The Uncertainties in the Critical Flow Functions Calculated with AGA8-DC92 and GERG-2008

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# Abstract

In this paper, the uncertainties in the critical flow functions (CFFs) calculated with the AGA8-DC92 and the GERG-2008 equations of state (EOSs for compression factor) were estimated. To this end, thermodynamic properties such as enthalpy, entropy, compression factor, and speed of sound, which are used in calculating CFF, were expressed in the form of dimensionless Helmholtz free energy and its derivatives. In order to identify the influence of the uncertainty in compression factor on CFF, the form of Helmholtz free energy for each EOS and its derivatives were modified to have a deviation corresponding to a deviation (i.e., uncertainty) in compression factor under each flow condition. For each independent uncertainty component of CFF, both a model and a method to estimate the uncertainty contribution were developed. As a result, the uncertainty in CFF is seen to increase with the increase in pressure, to increase with the increase in calorific value of a gas, and to show almost no change due to temperatures and the EOSs. The estimated uncertainties (at *k* = 2) in the CFFs calculated with both EOSs were 0.044 – 0.064 %, 0.049 – 0.069 %, and 0.055 – 0.075 % at stagnation pressures of 1, 5, and 8 MPa, respectively, and stagnation temperatures from 288 K to 300 K.

# 1. Introduction

In Korea, the construction of a closed-loop facility utilized for the calibration of high-pressure gas meters is underway. The facility employs a gravimetric system as the primary standard and a bank of critical flow Venturi nozzles (CFVNs) as the secondary standard; therefore, it is necessary to estimate the uncertainty in the critical flow function (CFF) appearing in the nozzle flow equation.

However, the calculation method of the uncertainty in CFF seems to be not yet established. Until the early 2000s, the uncertainty in CFF was regarded as nearly the same as an uncertainty of equation of state (EOS for compression-factor) in speed of sound (SoS), so that 0.1 % (at *k* = 2), which is the uncertainty of AGA8-DC92 EOS (AGA8) [1] in SoS for temperatures down to 250 K and pressures up to 5 MPa, was applied as the uncertainty in CFF. However, ISO-9300 [2], published in 2005, presented the uncertainty in the CFF calculated with AGA8 as 0.05 % without mentioning any flow conditions. The NIST started to apply the uncertainty in CFF of 0.061 % [3] instead of 0.11 %, where the value of 0.11 % is the existing uncertainty in CFF for one-dimensional gas flow. These have occurred since the mid 2000s, which has caused confusion about the uncertainty in CFF. In particular, the uncertainty in CFF of 0.05 % proposed by ISO-9300 is highly doubtful because it could be generated by only the uncertainty of AGA8 in compression-factor at high pressures. Therefore, this study aimed to establish a method of estimating the uncertainty in CFF, which is not yet clearly established. This study also estimated the uncertainties in the CFFs calculated with AGA8 and GERG-2008 EOS (GERG-2008) [4, 5] based on the above estimation method.

# 2. Critical flow function

*2.1 Definition of CFF*

Under real gas flow conditions, the equation for critical mass flow-rate passing through a CFVN can be expressed in two ways:

 (1)

. (2)

From the Equation (1) and (2), CFF is defined as Equation (3):

 . (3)

*2.2 The basis for computation of CFF*

A CFVN operates at maximum mass flow-rate or critical mass flow-rate. The gas flow speed at the nozzle throat is the speed of sound. This condition allows the calculation of the critical mass flux (CMF), , in Equation (3). Under the assumption that the gas flow is one-dimensional and that the entropy of the gas in the nozzle throat is the same as the entropy of the gas at the nozzle inlet or stagnation conditions:

|  |  |  |
| --- | --- | --- |
| Nomenclature | | |
| cross-sectional area of Venturi nozzle throat (m2)  , , , , etc. in Eq.(9) : see ISO 20765-1 [6]  , , , etc. in Eq.(10): see ISO 20765-2 [7]  discharge coefficient of Venturi nozzle  critical flow function for one-dimensional flow of real gas  *CV* calorific value of a gas  residual part of  *H* specific enthalpy (J/kg)  M molar mass of a gas (kg/kmol)  *p* pressure (MPa or Pa)  critical mass flow-rate  *R* gas constant *R* = 8 314.472 J/ kmol-K  *R*\* gas constant *R*\* = 8 314.51 J/kmol-K  *R*u  universal gas constant, *R*u = *R*\* for AGA8,  *R*u = *R* for GERG-2008  adjusting factor for (-)  *S* specific entropy (J/kg-K)  *T* temperature (K)  *u* uncertainty  critical flow velocity at Venturi nozzle throat (m/s)  speed of sound at Venturi nozzle throat conditions (m/s)  *Z* compression-factor of a gas, *Z* = (-)  critical flow velocity at Venturi nozzle throat (m/s) |  | speed of sound at Venturi nozzle throat conditions (m/s)  *X* gas composition vector  mole fraction  *Z* compression-factor of a gas, *Z* = (-)  *Z’* Modified compression-factor  uncertainty or deviation in compression factor,  *Greek*  reduced Helmholtz free energy (-)  difference or deviation  reduced density, see ISO 20765-1, 2[6,7]  virtual deviation of reduced density (-)  molar density (kmol/m3)  mass density (kg/m3)  inverse reduced temperature, see ISO 20765- 1, 2 [6,7 ]  *Subscripts*  Venturi nozzle throat  stagnation conditions  partial derivative with respect to molar reduced density  partial derivative with respect toinverse reduced temperature |
|  | | |

, (4)

 (5)

where flow velocity at the nozzle is sonic,

, (6)

 (7)

The conditions described by Equations (4) through (7) provide the basis for computation of CFF or CMF for CFVN.

# 3. Methodology

*3.1 Sources of the uncertainty in CFF*

Sources of the uncertainty in CFF are the uncertainty in compression-factor at stagnation conditions ()), the uncertainty in compression-factor at the nozzle throat conditions (), the uncertainty in SoS at the nozzle throat conditions (), and the uncertainty in function for ideal gas isobaric heat capacities. Among them, the uncertainty in function for ideal gas isobaric heat capacities is not needed to be considered because the accurate Jaeschke [8] model is normally used in CFF calculation, and most of the uncertainty is already included in the uncertainty in SoS. Among the other sources, although a correlation is present between andin CFF calculation, the percentage contribution of to the total uncertainty in is very small (max. 3 %), which is negligible. Therefore, ), , and were regarded as independent uncertainty components in this study.

*3.2 Modified Helmholtz free energy*

In order to identify the influence of the uncertainty in compression-factor on CFF, the form of Helmholtz free energy for each EOS should be modified to have a deviation corresponding to a deviation (i.e., uncertainty) in compression-factor under each flow condition. Before doing this, a type of the EOS shall be changed. In this process, note that a deviation in compression-factor shall be expressed as a form of density deviation in the Helmholtz free energy. By considering this, a type of EOS for compression-factor is changed as follows:

, (8)

where represents the virtual deviation in reduced density () corresponding to a deviation in compression -factor *Z* and approaches zero with approaching to zero. It is almost constant with respect to (i.e., ) except near . Once Equation (8) is divided by and integrated with respect to , the modified Helmholtz free energy (MHFE) for each EOS can be expressed as Equations (9) and (10) (ideal gas terms included). One problem while calculating CFF using the MHFE and its five derivatives is that Equation (11) is not satisfied.

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| **Modified Helmholtz free energy**  For the AGA8-DC92 EOS:  For the GERG-2008 EOS:  Each equation is the same as the original Helmholtz free energy [6-7] if = 0. |
|  |

(11)

A value of Equation (11) has a value between -1.16 and -1.00 in the range of stagnation pressure of up to 8 MPa. In order to correct this error, instead of , where is the adjusting factor, is applied only to the partial derivative related to compression-factor to establish Equation (11). Thus, the MHFE and its five derivatives including deviation can be expressed as , , , , , and .

*3.3 Relative sensitivity coefficient with respect to compression-factor*

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 1**: Test gas compositions | | | | | | | | | | | |
| Component | Component mole percent for indicated gas (mol %) | | | | | | | | | | |
| Gas AI-N | Gas B | Gas BI-N | Gas BI-C | Gas C | Gas D | Gas EI | Gas EI-1 | Gas FI-C | Gas GI | Gas HI |
| Methane  Ethane  Propane  i-Butane  n-Butane  i-Pentane  n-Pentane  n-Hexane  Nitrogen  Carbon dioxide | 90.2186  4.8871  1.3510  0.2383  0.2859  0.0191  0.0000  0.0000  3.0000  0.0000 | 93.9710  3.9730  1.2833  0.2685  0.2960  0.0151  0.0000  0.0000  0.1931  0.0000 | 84.6625  7.0376  2.2731  0.4756  0.5243  0.0268  0.0000  0.0000  5.0000  0.0000 | 91.4401  4.8063  1.5524  0.3248  0.3581  0.0183  0.0000  0.0000  0.0000  1.5000 | 93.0700  4.4900  1.5300  0.3300  0.3600  0.0200  0.0000  0.0000  0.2000  0.0000 | 92.1888  4.9858  1.7948  0.3931  0.4202  0.0186  0.0000  0.0000  0.1987  0.0000 | 85.9063  8.4919  2.3015  0.3486  0.3506  0.0509  0.0480  0.0000  1.0068  1.4954 | 88.5632  5.6618  2.3727  0.3594  0.3614  0.0525  0.0495  0.0000  1.0379  1.5416 | 87.9769  6.6775  2.6292  0.5808  0.6111  0.0246  0.0000  0.0000  0.0000  1.5000 | 96.5222  1.8186  0.4596  0.0977  0.1007  0.0473  0.0324  0.0664  0.2595  0.5956 | 90.6724  4.5279  0.8280  0.1037  0.1563  0.0321  0.0443  0.0393  3.1284  0.4676 |
| Calorific value  (MJ/(n)m3) | 41.51 | 42.35 | 42.35 | 42.36 | 42.77 | 43.19 | 43.63 | 42.81 | 44.03 | 40.79 | 40.75 |

Table 2 shows the calculation result of the relative sensitivity coefficient (RSC) [10-11] of CFF with respect to compression-factor, which means that percentage change in CFF produced by a one percent change in compression-factor, using the MHFE and its derivatives. The result covers the stagnation temperature range from 288 to 300 K and the stagnation pressure range from 1 to 8 MPa, which are the typical operation conditions of a CFVN. The calculation result shows that the RSC increases linearly with the increase in pressure, and the RSC to *Z*nt is approximately twice the RSC to *Z*o. The RSC between the EOSs has no difference at all and little difference between Gas C and Gas EI in Table 1, whose large difference in gas composition is revealed.

The difference in the RSC with the changes in temperature is also minimal. Based on the above results,

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 4:** **Table 2:** The relative sensitivity coefficients of CFF with respect to and with flow conditions and gas compositions  (a) Reference values of CFF | | | | | | | |
| EOS | | *T*o = 288 K/293K/300 K | | | | | |
| *p*o = 1 MPa | | *p*o = 5 MPa | | *p*o = 8 MPa | |
| Gas C | Gas EI | Gas C | Gas EI | Gas C | Gas EI |
| AGA8-DC92 | | 0.674 903  0.674 137  0.673 095 | 0.674 044  0.673 201  0.672 061 | 0.713 264  0.709 934  0.705 692 | 0.717 320  0.713 454  0.708 577 | 0.749 842  0.743 333  0.735 307 | 0.760 487  0.752 559  0.742 932 |
| GERG-2008 | | 0.674 873  0.674 115  0.673 083 | 0.673 972  0.673 143  0.672 018 | 0.713 112  0.709 826  0.705 629 | 0.717 003  0.713 219  0.708 422 | 0.749 555  0.743 114  0.735 209 | 0.759 838  0.752 121  0.742 688 |
| (b) The relative sensitivity coefficient with respect to | | | | | | | |
| AGA8-DC92 | -0.13  -0.13  -0.12 | | -0.12  -0.12  -0.12 | -0.17  -0.17  -0.16 | -0.17  -0.17  -0.16 | -0.22  -0.21  -0.20 | -0.23  -0.22  -0.21 |
| GERG-2008 | -0.13  -0.13  -0.12 | | -0.12  -0.12  -0.12 | -0.17  -0.17  -0.16 | -0.17  -0.17  -0.16 | -0.22  -0.21  -0.20 | -0.23  -0.22  -0.21 |
| (c) The relative sensitivity coefficient with respect to | | | | | | | |
| AGA8-DC92 | +0.25  +0.24  +0.24 | | +0.25  +0.24  +0.24 | +0.33  +0.32  +0.31 | +0.33  +0.32  +0.31 | +0.40  +0.39  +0.37 | +0.42  +0.40  +0.38 |
| GERG-2008 | +0.25  +0.24  +0.24 | | +0.25  +0.24  +0.24 | +0.33  +0.32  +0.31 | +0.33  +0.32  +0.31 | +0.40  +0.39  +0.37 | +0.42  +0.40  +0.38 |
|  | | | | | | | |

a model for the uncertainty contribution from each compression-factor can be expressed as follows from the RSCs at the temperature of 293 K given in Table 2.

 (12)

 (13)

*3.4 Relative sensitivity coefficient with respect to SoS*

The RSC of CFF with respect to SoS can be calculated if there are two accurate EOSs. This is because the RSC can be calculated from the relative SoS deviation in the nozzle throat conditions between two EOSs and the relative CFF deviation between two EOSs. The EOSs used in this study were GERG-2008 and AGA8, and all of the reference values of the CFF, nozzle throat conditions, compression-factors, and SoS were calculated using GERG-2008.



Figure : The SoS deviation versus enthalpy-difference deviation

Figure 1 represents the enthalpy-difference deviation () with regard to the corresponding SoS deviation (between the two EOSs in order to verify whether the uncertainty in enthalpy, which is in the range of 0.2 – 1.5 % [4-7], can affect the uncertainty in CFF. In CFF calculation, enthalpy term appears as a form of enthalpy-difference (). As shown in the Figure 1, the SoS deviation and the enthalpy-difference deviation are linearly proportional, passing through the origin. From this, the enthalpy-difference deviation is fully dependent on the SoS deviation. As a result, the uncertainty in enthalpy is not needed to be considered additionally in the estimation on CFF uncertainty.

In order to derive the RSC of CFF to SoS accurately, effects of the compression-factors included in the SoS deviation and CFF deviation between the two EOSs (more accurately, effects of the difference in compression-factor between the two EOSs) shall be excluded. This is due to an error that cannot be ignored, which can occur if a relationship (i.e., the contribution to the uncertainty in CFF from SoS) between the SoS deviation and the CFF deviation is derived without removing effects of compression-factors. This is because the RSC to *Z* is up to 0.42, as shown in Table 2, and RSC of SoS () to *Z*nt is from 0.89 to over 1.0; note that this RSC differs from RSC of SoS () to *Z*nt in CFF calculation. To eliminate these errors, the effects of the compression-factors on CFF deviation were removed by using Equations (12) and (13) after calculating the difference in compression-factor ( between the two EOSs at the nozzle throat conditions and the difference in compression-factor ( between the two EOSs at the stagnation conditions. The effect of the compression-factor on the SoS () deviation was removed via a value produced by multiplying the difference in compression-factor ( between the two EOSs by RSC of SoS to *Z*nt. Since most of the effects of compression-factors on the CFF deviation and the SoS deviation were removed, no other uncertainty components were considered.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 3:** Test results of Equation (14) | | | | | | | | | | | |
|  | Gas AI-N | Gas B | Gas BI-N | Gas BI-C | Gas C | Gas D | Gas EI | Gas EI-1 | Gas FI-C | Gas GI | Gas HI |
| RSC from EOSs, (A) | 0.40 | 0.46 | 0.46 | 0.46 | 0.48 | 0.50 | 0.51 | 0.50 | 0.58 | 0.35 | 0.35 |
| RSC from Eq. (14),(B) | 0.40 | 0.46 | 0.46 | 0.46 | 0.49 | 0.51 | 0.54 | 0.49 | 0.57 | 0.35 | 0.35 |
| Dev. (%), (B/A-1) | 0.0 | 0.0 | 0.0 | 0.0 | 2.1 | 2.0 | 5.9 | -2.0 | -1.7 | 0.0 | 0.0 |

Figure 2 shows the entire range of the uncertainty contribution from SoS found in this study. The main point of interest in SoS deviations is 0.1 %; this is due to the uncertainties of the two EOSs in SoS, both of which are 0.1 % at most operation conditions of a CFVN. The range of the RSC of CFF to , CFF (%)/(%), is 0.35 – 0.58 at the SoS deviation of 0.1 %, which indicate a significant difference between gas compositions. In order to identify the causes of the differences, investigating the calorific values given in Table 1, it is seen that the RSC increases with the increase in calorific value. Based on this result, a model for the uncertainty contribution from SoS can be expressed as Equation (14). In Table 3, comparison results of RSCs calculated from Equation (14) and the EOSs are presented, which all show deviation of within 5.9 %.



Figure 2: The uncertainty contribution from with gas compositions

 (14)

# 4. Estimation of the uncertainty in CFF

In Chapter 3, all RSCs with respect to CFF uncertainty sources were calculated. Therefore, if the uncertainty in compression-factor at stagnation conditions, the uncertainty in compression-factor at nozzle throat conditions, and the uncertainty in SoS at nozzle throat conditions are known, the uncertainty contribution from each uncertainty component can be calculated. Furthermore, the total uncertainty in CFF can be evaluated by combining all the contributions.

The typical operation conditions of a CFVN are in the temperature range from 288 K to 300 K at pressures up to 8 MPa under stagnation conditions and in the temperature range from 246 K to 257 K at pressures up to 4.3 MPa under nozzle throat conditions. For temperatures from 250 K to 350 K at pressures up to 9.3 MPa and typical natural gases of pipeline quality, the uncertainties of both AGA8 and GERG-2008 in compression-factor are 0.1 % [1, 4], and the uncertainties of GERG-2008 and AGA8 in SoS are 0.1 % [4] and 0.1 to 0.8 % [6, 9], respectively. However, since the SoS in CFF computation is calculated at the nozzle throat conditions, a pressure where the SoS is calculated does not exceed 5 MPa under which the uncertainties of both EOSs in SoS are 0.1 %. Based on these facts, no difference of the uncertainties of both EOSs in compression-factor, as well as no difference of the uncertainties of both EOSs in SoS, is found at the typical operation conditions of a CFVN. Therefore, no difference of the uncertainties in the CFFs calculated with both EOSs is found.

In order to estimate the uncertainty in CFF by extending the temperature down to 246 K (the corresponding stagnation temperature is 288 K), which is the aforementioned lower temperature limit, the uncertainties in compression-factor and SoS will be known at the above temperature. However, the problem is that related literatures do not specify the uncertainties in compression-factor and SoS at temperatures below 250 K. To overcome this limitation, the uncertainties of AGA8 and GERG-2008 in compression-factor and SoS for various binary-mixtures and temperatures below 250 K at pressures up to 5 MPa were checked, and the results showed that the uncertainties of both EOSs in compression-factor and SoS were 0.1 % at temperatures around 246 K and pressures up to 5 MPa [4–5]. Furthermore, for pressures up to 4.3 MPa, the relative compression-factor deviation and relative SoS deviation between the two EOSs at temperature 246 K showed very little difference (within 0.03 %) with the corresponding deviations at temperature 250 K. Based on the results, the uncertainties of both EOSs in compression-factor and SoS were regarded to remain at 0.1 % at temperatures around 246 K and pressures up to 4.3 MPa. Because of this, a stagnation pressure range in this study was limited up to 8 MPa, which correspond to the nozzle throat pressure of 4.3 MPa. Note that when the cricondentherm of a gas is close to 246 K, like Gas EI (refer to Figures 3 and 4), it is very difficult to cite the uncertainties in compression-factor and SoS from related literatures. This is because the experimental data for the two thermodynamic properties do not exist at close to two-phase zone, resulting in an estimating range of CFF uncertainty that cannot be extended down to temperature 246 K or even can be reduced to temperatures above 250 K.

The uncertainties (at *k* = 2) in CFF estimated using Equations (12) and (13) of the models of uncertainty contribution from compression-factors, Table 3 or Equations (14) of the model of uncertainty contribution from SoS were 0.044 – 0.064 %, 0.049 – 0.069 %, and 0.055 – 0.075 % at the stagnation pressures of 1, 5, and 8 MPa, respectively and stagnation temperatures from 288 K to 300 K.



Figure 3: SoS difference between the two EOSs for Gas EI



**Figure 4:** Phase envelopes for the Gas C and Gas EI

# 7. Conclusion

The uncertainties in the critical flow functions calculated with the AGA8-DC92 and the GERG-2008 equations of state were estimated. To this end:

1) the thermodynamic properties, such as enthalpy, entropy, compression factor, and speed of sound, which are used in calculating CFF, were expressed in the form of dimensionless Helmholtz free energy and its derivatives;

2) in order to identify the influence of the uncertainty in compression factor on CFF, the form of Helmholtz free energy and its five derivatives for each EOS was modified to have a deviation corresponding to an uncertainty in the compression-factor under each flow condition;

3) for each independent uncertainty component of CFF, a model and a method were developed to estimate the uncertainty contribution;

4) as a result, the uncertainties in CFF are seen to increase with the increase in pressure, to increase with the increase in calorific value, and to show almost no change with temperature and EOSs;

5) the estimated uncertainties in the CFFs calculated with the two EOSs were 0.044 – 0.064 %, 0.049 – 0.069 %, and 0.055 – 0.075 % at the stagnation pressures of 1, 5, and 8 MPa, respectively, and stagnation temperatures from 288 K to 300 K.

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